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Discriminating a gravitational-wave background from instrumental noise using time-delay interferometry

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Abstract
The first in-flight Doppler tracking measurements performed by LISA will be used to assess its detector performance. The existence of a strong stochastic background of gravitational radiation from many close binary systems in our Galaxy will prevent us from identifying the interferometer noise level in the frequency band 0.1–8 mHz. This in turn will prevent us from making an unambiguous detection of the background itself. By implementing the proposed multiple Doppler readouts, it is possible to generate several observables that are independent of laser phase fluctuations, and at the same time have different couplings to gravitational waves and to the various LISA instrumental noises. Comparison, for example, of the Michelson interferometer observable with the fully symmetric Sagnac data-type allows discrimination between a confusion-limited gravitational-wave background and instrumental noise.

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1. Introduction

LISA will detect and study, in the millihertz frequency band, a wealth of astrophysical sources of gravitational waves. With its three spacecraft, each carrying lasers, beamsplitters, photodetectors and drag-free proof masses on each of their two optical benches, LISA will have the capability of measuring six time series of Doppler shifts of the one-way laser beams between spacecraft pairs, and six shifts between adjacent optical benches on each spacecraft. By linearly combining, with suitable time delays, these 12 data sets, it will be possible to cancel the otherwise overwhelming phase noise of the lasers ($\Delta\nu/\nu \simeq 10^{-13}$) to a level $h \simeq \Delta v/c \simeq 10^{-23}$. This level is set by the buffeting of the drag-free proof masses inside each optical bench, and by the shot noise at the photodetectors [1–3].

LISA is expected to detect individual signals from galactic binaries. Particularly at low Fourier frequencies (say 0.1–8 mHz), however, it is expected that there will be many sources radiating within each Fourier resolution bin [4–6]. These latter signals will not be detectable individually, forming a continuum which could be confused with instrumental noise. The level of this continuum is uncertain but could be in the range 10^{-20}–10^{-23} [4–6]. A measurement of the amplitude and Fourier frequency dependence of this background, and its variation with position on the sky, will confirm or disprove estimates of galactic binary system populations [7]. Since these galactic binary populations are virtually guaranteed, the detection of their signals could be the first direct detection of gravitational waves.

For this measurement it is very desirable that competing proof-mass or other instrumental noises be both characterized and calibrated before flight, and measured in the actual flight configuration while data are being taken. In contrast to Earth-based, equi-arm interferometers for gravitational wave detection, LISA will have multiple readouts, and the Doppler data they generate can be combined differently to give measurements not only insensitive to laser phase fluctuations and optical bench motions, but also with different sensitivities to gravitational waves and to the remaining system noise [2, 3, 8].

This paper briefly discusses two laser-and-optical-bench-noise-free combinations of the LISA readouts, previously denoted ζ (Sagnac) and X (Michelson), that have very different responses to the gravitational-wave background but comparable responses to instrumental noise sources. Here we give an outline of the paper.

In section 2, after giving a brief summary of the time-delay interferometric technique [2,3,8], we compare the sensitivities of the interferometric combinations derived in our previous publications to the theoretical estimates of the intensity of the galactic stochastic background. We show that the symmetric combination of the one-way data, ζ, is largely insensitive to the gravitational wave signal, and provides information about the noise that affects the unequal-arm Michelson interferometer response, X. In section 3 we point out that ζ can be regarded as a *gravitational wave shield* for a space-based detector of gravitational waves. It allows one to measure the noise affecting the interferometer response, thereby uniquely assessing the presence or absence of a gravitational-wave background in the frequency band 0.1–8 mHz accessible by LISA. In section 4 we present our final remarks and conclusions.

2. The Sagnac and Michelson interferometers

The six Doppler beams exchanged between the LISA spacecraft imply the six Doppler readouts y_{ij} $(i, j = 1, 2, 3)$ recorded when each transmitted beam is mixed with the laser light at the receiving optical bench. Delay times for light travel between the spacecraft must be carefully accounted for when combining these data. Six further data streams, denoted z_{ij} $(i, j = 1, 2, 3)$, are generated internally to monitor both lack of rigidity and laser synchronization between the independent optical benches at each spacecraft. We use all the conventions and definitions of [3]. The combination ζ uses all the Doppler data symmetrically

$$\begin{aligned}\zeta = {} & y_{32,2} - y_{23,3} + y_{13,3} - y_{31,1} + y_{21,1} - y_{12,2} \\ & + \tfrac{1}{2}(-z_{13,21} + z_{23,12} - z_{21,23} + z_{31,23} - z_{32,13} + z_{12,13}) \\ & + \tfrac{1}{2}(-z_{32,2} + z_{12,2} - z_{13,3} + z_{23,3} - z_{21,1} + z_{31,1}). \end{aligned} \tag{1}$$

The comma notation indicates time-delays along the arms of the three-spacecraft configuration

$$y_{32,2} \equiv y_{32}(t - L_2), \tag{2}$$

and so forth (units in which $c = 1$).

The transfer functions of ζ to instrumental noises and to gravitational waves were calculated in [3]. Using current specifications for random velocities expected for the six drag-free proof masses, for fluctuations due to shot noise at the readouts and for beam-pointing noise,

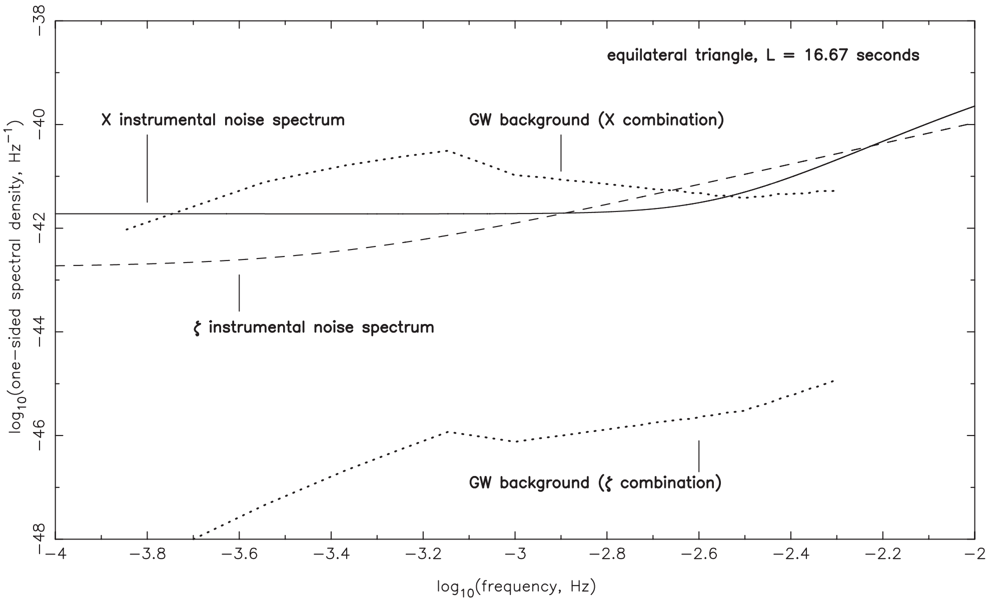

Figure 1. Fractional Doppler frequency instrumental noise power spectra for data combinations X and ζ. These are derived from the transfer functions for X and ζ to instrumental noises and the nominal LISA spectra for individual proof mass noise (3×10^{-15} (m s^{-2}) Hz$^{-1/2}$) and one-way optical path noise (20×10^{-12} m Hz$^{-1/2}$), converted to fractional Doppler spectra. An equilateral triangle configuration with arm length of 16.67 s is assumed. Also plotted are the spectral responses of X and ζ to the smaller of the stochastic gravitational-wave backgrounds discussed by Bender and Hils [4], and Hils [5]. Using ζ to measure on-orbit instrumental noise allows a gravitational-wave background in X to be either uniquely determined or bounded.

the expected noise power spectrum entering ζ can then be computed. The resulting instrumental noise power spectrum for ζ is shown in figure 1. Also shown there is the computed power spectrum of ζ, averaged over the sky and elliptical polarization states, that would result from a candidate stochastic background originated by an ensemble of galactic binary systems, based in figure 2 of Bender and Hils [4].

The laser-and-optical-bench-noise-free combination, X, only requires four data streams. If low-noise optical transponders can be used at spacecraft 2 and 3, then only the two readouts on board spacecraft 1 are needed [1, 2]. This combination is equivalent to an (unequal arm) Michelson interferometer. In general it is given by

$$X = y_{32,322} - y_{23,233} + y_{31,22} - y_{21,33} + y_{23,2} - y_{32,3} + y_{21} - y_{31} + \tfrac{1}{2}(-z_{21,2233} + z_{21,33} + z_{21,22} - z_{21}) + \tfrac{1}{2}(+z_{31,2233} - z_{31,33} - z_{31,22} + z_{31}). \quad (3)$$

The expected instrumental noise power spectrum in X, using equation (3), appropriate transfer functions [3], and expected shot and proof mass noise spectra [1], are shown in figure 1. Also shown is one anticipated galactic binary confusion spectrum [4], which would be observed in X. Comparison of X and ζ allows the background to be discriminated from instrumental noise.

3. Discriminating a stochastic background from instrumental noise

LISA will have an equilateral configuration with $L_1 = L_2 = L_3 = L = 16.67$ s. In the frequency band of interest (0.1–8 mHz), the expressions for the Fourier transforms of the gravitational wave signals $\widetilde{X}^{gw}(f)$, $\widetilde{\zeta}^{gw}(f)$ and the power spectral densities of the system noises in X and ζ, $S_{X^{noise}}(f)$, $S_{\zeta^{noise}}(f)$, can be Taylor-expanded in the dimensionless quantity fL. The first non-zero terms are

$$\widetilde{X}^{gw}(f) \simeq 2(2\pi \mathrm{i} fL)^2 \left[\hat{n}_3 \cdot \widetilde{\boldsymbol{h}}(f) \cdot \hat{n}_3 - \hat{n}_2 \cdot \widetilde{\boldsymbol{h}}(f) \cdot \hat{n}_2\right], \tag{4}$$

$$\widetilde{\zeta}^{gw}(f) \simeq \tfrac{1}{12}(2\pi \mathrm{i} fL)^3 \big[(\hat{k} \cdot \hat{n}_1)(\hat{n}_1 \cdot \widetilde{\boldsymbol{h}}(f) \cdot \hat{n}_1) + (\hat{k} \cdot \hat{n}_2)(\hat{n}_2 \cdot \widetilde{\boldsymbol{h}}(f) \cdot \hat{n}_2) \\ +(\hat{k} \cdot \hat{n}_3)(\hat{n}_3 \cdot \widetilde{\boldsymbol{h}}(f) \cdot \hat{n}_3)\big], \tag{5}$$

$$S_{X^{noise}}(f) \equiv S_{X^{proof\,mass}}(f) + S_{X^{optical\,path}}(f) \\ \simeq 16\,[S_1(f) + S_{1^*}(f) + S_3(f) + S_{2^*}(f)]\,(2\pi fL)^2 \\ +4\,[S_{32}(f) + S_{23}(f) + S_{31}(f) + S_{21}(f)]\,(2\pi fL)^2 \tag{6}$$

$$S_{\zeta^{noise}}(f) \simeq [S_1(f) + S_2(f) + S_3(f) + S_{1^*}(f) + S_{2^*}(f) + S_{3^*}(f)]\,(2\pi fL)^2 \\ + [S_{32}(f) + S_{23}(f) + S_{31}(f) + S_{21}(f) + S_{13}(f) + S_{12}(f)], \tag{7}$$

where we have denoted by $S_{X^{proof\,mass}}(f)$, and $S_{X^{optical\,path}}(f)$ the aggregate contributions to the power spectrum of the noise in the response X from the proof mass and optical path noises, respectively. The expressions in square brackets in equations (4) and (5) incorporate LISA antenna responses [8], and are of the same order of magnitude. The proof mass Doppler noise spectra $S_i(f)$, $S_{i^*}(f)$ $(i = 1, 2, 3)$ will be designed to a nominal power spectral level $S^0(f) = 2.5 \times 10^{-48}[f/1\ \mathrm{Hz}]^{-2}\ \mathrm{Hz}^{-1}$ [1, 3], while the optical path noise spectra $S_{ij}(f)$ $(i, j = 1, 2, 3, i \neq j)$, which include shot noises at the photo detectors and beam pointing noise [3], are expected to be equal to a nominal spectrum $S^1(f) = 1.8 \times 10^{-37}[f/1\ \mathrm{Hz}]^2$. The spectra for fractional frequency fluctuations correspond to an acceleration spectrum of $3.0 \times 10^{-15}\ \mathrm{m\ s^{-2}\ Hz^{-1/2}}$, and optical path length noise spectrum of $20 \times 10^{-12}\ \mathrm{m\ Hz^{-1/2}}$, respectively. Both these noise sources will be estimated before launch, but could be larger when the in-orbit data is taken.

First, consider the responses to the gravitational wave signal, given in equations (4) and (5). At $f = 10^{-3}$ Hz, for instance (where $2\pi fL \simeq 10^{-1}$) the absolute value of the coefficient in front of the squared-brackets in the ζ response (equation (5)) is about three orders of magnitudes smaller than the corresponding coefficient given in the expression for X (equation (4)). The power spectral densities of the noises due to the proof masses and the optical-path noise (equations (6) and (7)) will only differ at most by an order of magnitude. We conclude that in this lower-frequency range the LISA Sagnac response, ζ, can be used as a *gravitational wave shield*. In what follows we will ignore the gravitational-wave background contribution to ζ.

To take quantitative advantage of this property of ζ, consider the observed power spectral densities of X and ζ

$$S_X^{obs}(f) = S_{X^{gw}}(f) + S_{X^{proof\,mass}}(f) + S_{X^{optical\,path}}(f) \tag{8}$$

$$S_\zeta^{obs}(f) = \frac{1}{16}\left[S_{X^{proof\,mass}}(f) + \frac{S_{X^{optical\,path}}(f)}{(\pi fL)^2}\right] + [S_{13}(f) + S_{12}(f)] \\ + [S_2(f) + S_{3^*}(f)]\,(2\pi fL)^2, \tag{9}$$

where in equation (9) we have written the power spectra of the noises in ζ in terms of the power spectra of the noises in X and of some remaining terms that are not present in X, to emphasize commonality of some noise sources. We suppose that the actual noise contributed by any one of the proof masses and optical-path noise sources will be greater than or equal to the design values, $S^0(f)$ and $S^1(f)$.

From equation (9), if the magnitude of the *measured* power spectral density of the response ζ is at its anticipated level $S_\zeta^{obs}(f) = 6S^0(f)(2\pi f L)^2 + 6S^1(f)$, then the level of the power spectral density of the noise entering into X is known. The spectrum

$$S_{X^{gw}}(f) = S_X^{obs}(f) - 64S^0(f)(2\pi f L)^2 - 16S^1(f)(2\pi f L)^2, \tag{10}$$

should then be attributed to a galactic binary background of gravitational radiation. In any event, the right-hand side of equation (10) is an upper bound to $S_{X^{gw}}$.

On the other hand, if the measured spectrum of ζ is above its anticipated design level, consider the following combination of the measured spectra:

$$\begin{aligned} S_X^{obs}(f) - 16S_\zeta^{obs}(f) = S_{X^{gw}} - 16\,[S_2(f) + S_{3^*}(f)]\,(2\pi f L)^2 - 16\,[S_{13}(f) + S_{12}(f)] \\ -16\,[S_{32}(f) + S_{23}(f) + S_{31}(f) + S_{21}(f)]\left[1 - (\pi f L)^2\right]. \end{aligned} \tag{11}$$

The coefficient of S_ζ^{obs} has been chosen so that the noise terms on the right-hand side are all now negative-definite and can thus be bounded from above by their design, or nominal, values $S^0(f)$ and $S^1(f)$, respectively. The result is a *lower* bound for observational discrimination of the gravitational-wave background spectrum

$$S_{X^{gw}}(f) \geqslant S_X^{obs}(f) - 16S_\zeta^{obs}(f) + 32(2\pi f L)^2 S^0(f) + 16[6 - (2\pi f L)^2]S^1(f). \tag{12}$$

Equations similar to (11) and (12) can be written for the other two interferometer combinations, Y and Z [3]. In those equations, there will be different mixes of cancelled and bounded noise sources, resulting, in general, in different gravitational wave spectrum lower bounds. We note moreover that such bounds result not only from using ζ with data combinations X, Y, and Z, but also with other data types [3] such as α, P, etc. If certain of the proof masses are significantly noisier than others, this can make certain of these bounding criteria preferable. As an example, the spectral difference $S_\alpha^{obs}(f) - 9S_\zeta^{obs}(f)$ has negative-definite noise and leads to the lower bound

$$S_{\alpha^{gw}}(f) \geqslant S_\alpha^{obs}(f) - 9S_\zeta^{obs}(f) + 32(2\pi f L)^2 S^0(f) + 48S^1(f). \tag{13}$$

4. Conclusions

The response of the Sagnac interferometer to a gravitational-wave signal is several orders of magnitudes smaller than that of the Michelson interferometer. In the frequency band of interest (0.1–8 mHz), however, the Sagnac response to the noise sources is of the same order of magnitude as that of the Michelson interferometer. As a consequence of these facts we have shown that it is possible to estimate the magnitude of the noise sources affecting the Michelson interferometer response in the low-frequency region of the accessible band by using the Sagnac interferometer. This in turn allows us to discriminate a gravitational-wave background of galactic origin from instrumental noise affecting the Michelson interferometer response.

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